

# An Optimal Transition Path for Controlling Greenhouse Gases

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Designing efficient policies to slow global warming requires an approach that combines economic tools with relations from the natural sciences. The dynamic integrated climate-economy (DICE) model presented here, an intertemporal general-equilibrium model of economic growth and climate change, can be used to investigate alternative approaches to slowing climate change. Evaluation of five policies suggests that a modest carbon tax would be an efficient approach to slow global warming, whereas rigid emissions- or climate-stabilization approaches would impose significant net economic costs.

Scientists have warned that the accumulation of carbon dioxide and other greenhouse gases (GHGs) is likely to lead to global warming and other significant climatic changes over the next century. Responding to growing concerns from scientific and environmental groups, governments have recently approved a framework treaty on climate change to monitor trends and national efforts, and this treaty formed the centerpiece of the Earth Summit held in Rio in June 1992 (1).

To date, the calls for stringent controls and the treaty negotiations have progressed more or less independently of economic studies of the costs and benefits of measures to slow greenhouse warming. Estimating the costs and benefits of these measures poses daunting problems for economists and other policy analysts, raising formidable issues of data, modeling, uncertainty, international coordination, and institutional design. Furthermore, the economic stakes are enormous, involving investments on the order of hundreds of billions of dollars a year to slow or prevent climate change.

Most early studies of the economics of climate change have focused on the cost of attaining a particular path for the reduction of GHG concentrations or emissions (2, 3). These studies have not addressed the more difficult issue of the damages averted by emissions reductions. A simple equilibrium cost-benefit framework for determining the optimal steady-state control of CO<sub>2</sub> and other GHGs concluded that the threat of greenhouse warming was sufficient to justify modest investments to slow the pace of climate change (4, 5).

This study presents the dynamic integrated climate-economy (DICE) model of global warming (6, 7). The DICE model is

an integrated model that incorporates the dynamics of emissions and economic impacts as well as the economic costs of policies to curb emissions.

## The DICE Model

The DICE model is a dynamic optimization model for estimating the optimal path of reductions of GHGs (8). The basic approach is to estimate the optimal path for both capital accumulation and reductions of GHG emissions in the framework of the Ramsey model of intertemporal choice (9, 10). The resulting trajectory can be interpreted as the most efficient path for slowing climate change given inputs and technologies; alternatively, the trajectory can be interpreted as a competitive market equilibrium in which externalities or spillover effects are corrected with the use of the appropriate social prices for GHGs.

In the DICE model, emissions include all GHGs but are most easily interpreted as CO<sub>2</sub>. Uncontrolled emissions make up a slowly declining fraction of gross output. Greenhouse-gas emissions, which accumulate in the atmosphere, can be controlled by an increase in the prices of inputs (such as energy) or outputs that are GHG-intensive. Climate change is represented by realized global mean surface temperature, which uses relations based on current climate models. The economic impacts of climate change are assumed to be increasing in the realized temperature increase.

In a more detailed derivation of the DICE model, the global economy is assumed to have an initial stock of capital and labor and a gradually improving technology. Population growth and technological change are exogenous, whereas capital accumulation is determined by optimization. In estimating the efficient paths for capital accumulation and emissions reduction, the DICE model treats the world as a single economic entity and analyzes the optimal

policy for the average individual (11).

The major choice faced by the economy in the DICE model is whether to consume goods and services, to invest in productive capital, or to slow climate change. This choice is represented by maximization of an objective function that is the discounted sum of the utilities of per capita consumption

$$\max_{\{c(t)\}} \sum_{t=1}^T U[c(t), P(t)](1 + \rho)^{-t} \quad (1)$$

Here,  $U$  is the level of utility or social well-being,  $c(t)$  is the flow of consumption per capita at time  $t$ ,  $P(t)$  is the level of population at time  $t$ , and  $\rho$  is the pure rate of social time preference. The objective function is then the discounted sum of the utilities of consumption,  $U[c(t), P(t)]$ , summed over the relevant time horizon from  $t = 1$  to  $t = T$ . The maximization is subject to two sets of constraints: first, a conventional set of economic constraints; and second, the specific set of emissions-climate-economy constraints.

*Economic constraints.* The first set of constraints are those relating to the growth of output known as the Ramsey model. The first equation is the definition of utility, which is equal to the size of population  $[P(t)]$  times the utility of per capita consumption  $U[c(t)]$ . Preferences are represented by a constant-elasticity-of-substitution utility function

$$U[c(t)] = P(t)\{[c(t)]^{1-\alpha} - 1\}/(1 - \alpha) \quad (2)$$

In this equation,  $\alpha$  is a measure of the social valuation of different levels of consumption called the rate of inequality aversion. When  $\alpha$  is 0, the utility function is linear and there is no social aversion to inequality; as  $\alpha$  gets larger, the social welfare function becomes increasingly egalitarian. In the experiments,  $\alpha$  is 1, which is the logarithmic or Bernoullian utility function (12).

Output  $[Q(t)]$  is given by a constant-returns-to-scale Cobb-Douglas production function in technology  $[A(t)]$ , capital  $[K(t)]$ , and labor, which is proportional to population

$$Q(t) = \Omega(t)A(t)K(t)^\gamma P(t)^{1-\gamma} \quad (3)$$

The elasticity of output with respect to capital is given by  $\gamma$ , whereas the term  $\Omega(t)$  relates to climatic impacts and will be described in Eq. 13.

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Gross output  $Q(t)$  can be devoted either to gross investment  $I(t)$  or to consumption  $C(t)$

$$Q(t) = C(t) + I(t) \quad (4)$$

whereas per capita consumption is

$$C(t) = C(t)/P(t) \quad (5)$$

Finally, the balance equation for the capital stock is

$$K(t) = (1 - \delta_K)K(t-1) + I(t) \quad (6)$$

where  $\delta_K$  is the rate of depreciation of the capital stock.

*Climate-emissions-damage equations.* The next set of constraints represents the linkages between the economy and climate change; it includes equations for emissions, concentrations, climate change, damages, and mitigation costs. These constraints pose major obstacles for modelers because they require the development of aggregate relations for a number of extremely complex natural and economic phenomena.

The first equation links GHG emissions to economic activity. In the analysis that follows, we convert each of the GHGs into its  $\text{CO}_2$  equivalent using a measure of that gas's total warming potential (the contribution of the gas to global warming summed over the indefinite future). Because 80% of the total warming potential is due to  $\text{CO}_2$ , parameters are generally drawn from studies of  $\text{CO}_2$ .

On the basis of historical data, the ratio of uncontrolled GHG emissions to gross output [ $\sigma(t)$ ] is assumed to decline exogenously at 1.25% per annum. Energy policies and other emissions policies can be used to reduce GHG emissions; emissions policies are represented by an emissions control rate,  $\mu(t)$ , which is the fractional reduction of emissions relative to the uncontrolled level. Hence, the emissions equation is

$$E(t) = [1 - \mu(t)]\sigma(t)Q(t) \quad (7)$$

where  $E(t)$  is GHG emissions and  $\sigma(t)$  is determined from historical data. The emissions control rate,  $\mu(t)$ , is determined by the optimization after 1990.

The next relation in the economy-climate nexus represents the accumulation of GHGs in the atmosphere. For the non- $\text{CO}_2$  GHGs, this involves estimating the atmospheric lifetimes or chemical transformations. Carbon dioxide accumulation and transportation is represented as a box model, in which each of the boxes is well mixed, and can be reduced to

$$M(t) = \beta E(t) + (1 - \delta_M) M(t-1) \quad (8)$$

where  $M(t)$  is  $\text{CO}_2$  concentrations relative to preindustrial times,  $\beta$  is the marginal atmospheric retention ratio, and  $\delta_M$  is the rate of transfer from the rapidly mixing reservoirs to the deep ocean. This equation is the GHG analog of the capital accumulation equation.

The inverse of the transfer rate,  $1/\delta_M$ , is the GHG turnover time and has been variously estimated to lie between 50 and 200 years for  $\text{CO}_2$ . We follow the Intergovernmental Panel on Climate Change (IPCC) (13) in taking a turnover time of 120 years (14). The marginal atmospheric retention rate,  $\beta$ , is estimated with the use of annual data on emissions and concentrations of  $\text{CO}_2$

$$M(t) - 0.9917 M(t-1) = 0.64 (\pm 0.15) E(t) \quad (9)$$

Note that  $0.9917 = 1 - \delta_M$ . For the sample period 1860 to 1985, this equation has an overall fit of  $R^2 = 0.803$  and a standard error of 0.519 GtC (billion tons of carbon). Equation 9 is used in the model.

The next step in the economy-climate relation concerns the link between the accumulation of GHGs and climate change. Climate modelers have developed a wide variety of approaches for estimating the impact of rising concentrations of GHGs on climate. On the whole, existing general circulation models (GCMs) are much too complex to be incorporated in economic models. In addition, most studies focus on equilibrium relations, whereas for economic analyses it is essential to understand the dynamics or transient properties of the response of climate to GHG concentrations.

The basic approach in this step involves the development of a small model that parameterizes the overall relation between GHG concentrations and the dynamics of climate change. For this purpose, the DICE model draws on Schneider and Thompson (15) for the basic equations. In this approach, the climate system is characterized by a multilayer system comprising the atmosphere, the mixed layer of the oceans, and the deep oceans

$$\begin{aligned} T_1(t) &= T_1(t-1) + (1/R_1) \\ &\quad \{F(t) - \lambda T_1(t-1) - (R_2/\tau_2) \\ &\quad [T_1(t-1) - T_2(t-1)]\} \\ T_2(t) &= T_2(t-1) + (1/R_2) \{(R_2/\tau_2) \\ &\quad [T_1(t-1) - T_2(t-1)]\} \end{aligned} \quad (10)$$

where  $T_i(t)$  is the temperature of layer  $i$  in period  $t$  (relative to 1900);  $i$  is 1 for the atmosphere and upper oceans and 2 for the deep oceans;  $F(t)$  is the radiative forcing in the atmosphere from GHGs (relative to 1900);  $R_i$  is the thermal capacity of the different layers;  $1/\tau_2$  is the transfer rate from the upper layer to the lower layer; and  $\lambda$  is the climate feedback parameter.

The next step is to find the appropriate numerical representation of the simplified climate model in Eq. 10. The parameters in Eq. 10 can be compared to transient runs from larger GCMs and to historical data.

The results of the models disagree by a wide margin, and the historical data are even further at variance with the climate models. The parameters in the DICE model are calibrated to a study by Schlesinger and Jiang (16). The DICE equations have an equilibrium temperature- $\text{CO}_2$  sensitivity of  $3^\circ\text{C}$  for a doubling of atmospheric  $\text{CO}_2$ , close to that of the scientific consensus (17). In addition, the adjustment time (the time required to reach  $1 - 1/e$  or 63% of the equilibrium temperature) is 19 years.

The next link in the economy-climate chain is to estimate the impact of climate change on human and natural systems. Assessment of the damages from greenhouse warming has proven extremely elusive (5). It has been estimated that the net economic damage from a warming of  $3^\circ\text{C}$  is likely to be 0.25% of national income (\$15 billion at 1992 prices and level of output) for the United States in terms of those variables that have been studied (5). Because this estimate excluded several areas that are inadequately studied, I adjusted it to 1% of total United States income to allow for these omitted factors. [By comparison, Cline's estimate for the United States was 1.1% of output for a warming of  $2.5^\circ\text{C}$  (18, 19).] Adjustments made for output composition in different countries raised the total impact to 1.3% of global output for all countries. In addition, there is evidence that the economic impact increases nonlinearly with climate change; the impact is here taken to be a quadratic function [Cline (19) estimated a power of about 1.3]. Therefore, the final relation between global temperature increase and income loss is

$$d(t) = 0.0133 [T(t)/3]^2 \quad (11)$$

where  $d(t)$  is the fractional loss of global output from greenhouse warming.

The final link in the economy-climate chain is the cost of reducing GHGs. This is one area that has been extensively studied and, although not without controversy, surveys indicate that the general slope and shape of the cost function seem relatively robust (17, 19, 20). These surveys found that after a reduction of GHG emissions by one-tenth, the cost curve rises sharply. A 50% reduction in GHG emissions is estimated to cost almost \$200 billion per year in the present global economy, or about 1% of world output. This estimate is understated to the extent that policies are inefficient or are implemented in a crash program and overstated to the extent that new, low-GHG technologies become available. The equation used in the model is

$$TC(t) = b_1 \mu(t)^{b_2} = 0.0686 \mu(t)^{2.887} \quad (12)$$

where  $TC(t)$  is the fractional cost to global output from GHG emissions control, and  $b_1$  and  $b_2$  are constants.

**Table 1.** Impact of alternative policies on consumption. The DICE model provides estimates of the discounted value of the utility of consumption from Eq. 1. The base value shows the total discounted value of consumption from 1990 on, whereas the columns on impacts show the difference between a particular policy and the no-controls policy. Values are given in 1989 U.S. dollars.

Case	Policy	Base value ( $\times 10^{12}$ )	Impact of program	
			Dollar difference ( $\times 10^9$ )	Percent difference
1	No-controls	731 694	0	0.000
2	Optimal policy	731 893	199	0.027
3	Stabilize emissions	726 531	-5163	-0.706
4	Stabilize climate	701 764	-29930	-4.091
5	Geoengineering	735 787	4093	0.559

Combining the cost and damage relations yields the climate factor,  $\Omega$ , in the production function

$$\begin{aligned} \Omega(t) &= [1 - b_1 \mu(t)^{b_2}] [1 + d(t)] \\ &= [1 - 0.0686 \mu(t)^{2.887}] \\ &\quad [1 + 0.00144 T(t)^2] \end{aligned} \quad (13)$$

**Projections.** Equations 1 through 13 can be solved by nonlinear optimization (21). Data on the major variables were collected for 3 years (1965, 1975, and 1985), whereas future periods were estimated by the calculations described above and in (21). Data on population, gross national product, consumption, and investment were obtained from existing data sources of the World Bank, the United Nations Educational, Scientific, and Cultural Organization (UNESCO), the Organization for Economic Cooperation and Development (OECD), and national governments.

Assumptions about future growth trends are as follows: The rate of growth of population was assumed to decrease slowly, stabilizing at 10.5 billion people in the 22nd century. The rate of growth of total factor productivity [the growth rate of  $A(t)$  in Eq. 3] was calculated to be 1.3% per annum in the period from 1960 to 1989. This rate was assumed to decline slowly over the coming decades. We calibrate the model by fitting the solution for the first three decades to the actual data for 1965, 1975, and 1985 and then optimizing for capital accumulation and GHG emissions in the future [see (7) for details].

### Policy Experiments

The DICE model has numerous applications, and we report here on an appraisal of five alternative policies toward global warming. First, in the no-controls policy, no steps are taken to slow or reverse greenhouse warming (although individuals would, of course, adapt to the changing climate). This policy has been followed for the most part by nations through 1989.

Second, the optimal policy undertakes

to construct economically efficient or optimal policies to slow climate change. This run maximizes the present value of economic welfare in Eq. 1 subject to all constraints. This policy can be thought of as one in which the nations of the world levy taxes or impose regulations that efficiently reduce GHG emissions in the period after 1990.

The third policy, emissions stabilization, is motivated by the proposals put forth by many governments that  $\text{CO}_2$  emissions be stabilized at 1990 levels; this was the policy that the United States rejected at the Earth Summit in Rio in June 1992. This target is implemented in the DICE model as a stabilization of the radiative equivalent of chlorofluorocarbon (CFC) and  $\text{CO}_2$  emissions at 1990 levels, where these are converted to a  $\text{CO}_2$ -equivalent basis. In quantitative terms, this represents an emissions limitation of 8.045 GtC equivalent of  $\text{CO}_2$  and CFCs per year. This policy has no particular scientific or economic merit, although it has the virtue of simplicity. Given a growing uncontrolled emissions path, emissions stabilization implies a growing percentage reduction of GHGs in the future.

A more ambitious approach, climate stabilization, attempts to slow climate change to a pace that will prevent major ecological impacts. One proposal is to slow the rate of temperature increase to  $0.1^\circ\text{C}$  per decade from 1950. This policy is, by my

calculation, unfeasible given the current buildup of GHGs. A feasible policy is to slow the GHG-induced global temperature increase to  $0.2^\circ\text{C}$  per decade after 1985, with an upper limit of a total increase of  $1.5^\circ\text{C}$  from 1900.

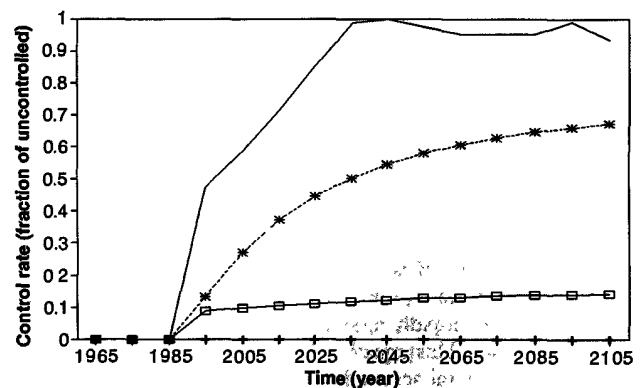
A final policy, geoengineering, would introduce a hypothetical technology that provides costless mitigation of climate change. This could occur, for example, if one of the geoengineering options proved technically feasible and environmentally benign. Two interesting proposals include shooting smart mirrors into space with 16-inch naval rifles or seeding the oceans with iron to accelerate carbon sequestration. [Several geoengineering solutions have extremely low economic costs compared to conventional mitigation techniques and can therefore be treated as costless; issues of geoengineering are discussed in depth in (17).] An alternative interpretation would be that the greenhouse effect has no harmful economic effects. This interpretation is useful as a baseline to determine the overall economic impact of greenhouse warming and of policies to combat warming.

### Calculations

As shown in Table 1, the optimal policy has a discounted net benefit of \$199 billion (1989 prices) relative to the no-controls policy. This number is absolutely large, although it is only 0.027% of the discounted value of consumption. More aggressive policies have negative net benefits. Emissions stabilization at 1990 rates has a net discounted cost of \$5.2 trillion, or 0.71% of discounted consumption. The ecological policy of stabilizing climate would require heavy investments, costing \$30 trillion in discounted income relative to the optimal policy, which amounts to 4.1% of the discounted value of consumption.

The overall economic impact of a costless geoengineering approach is equivalent to the estimated cost of climate change. Such a policy would have net benefits of \$4.1 trillion relative to the no-control pol-

**Fig. 1.** Greenhouse-gas control rate. Emissions in the DICE model are uncontrolled for the first three periods. The optimal control rate ( $\square$ ) shows the path that maximizes economic welfare. The uncontrolled path (+) has a control rate of 0. Emissions stabilization (\*) requires sharply increasing controls, whereas limiting temperature change to  $1.5^\circ\text{C}$  (—) requires virtual elimination of GHG emissions.



icy; this represents 0.56% of the discounted value of consumption.

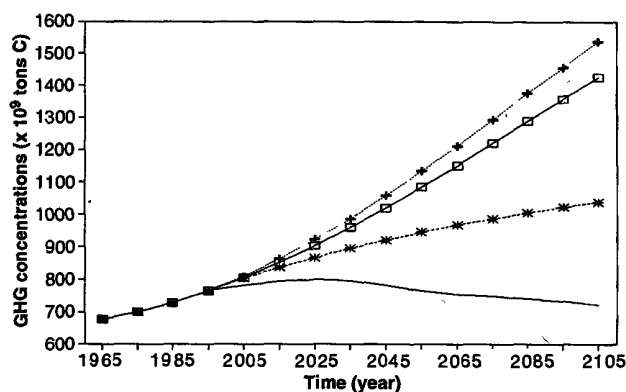
These numbers are mind-numbing in absolute size because they refer to the impact on global output over the indefinite future. On the other hand, with the exception of the policy of stabilizing climate, the numbers are modest relative to the total size of the global economy.

Emissions control rates differ greatly among the alternative policies (Fig. 1). In the optimal path, the rate of emissions reduction is approximately 10% of GHG emissions in the near future, rising to 15% late in the next century, whereas climate stabilization requires virtually complete elimination of GHG emissions. In the optimal control strategy, GHG concentrations are reduced by a little more than 100 GtC at the end of the next century (Fig. 2). With respect to mean global surface temperatures (Fig. 3), the optimal path shows a small decline (about 0.2°C by the end of the next century) in the growth rate of global temperatures relative to the uncontrolled path. Surprisingly, even draconian policies will slow climate change only modestly because of the momentum in the system from existing concentrations of GHGs.

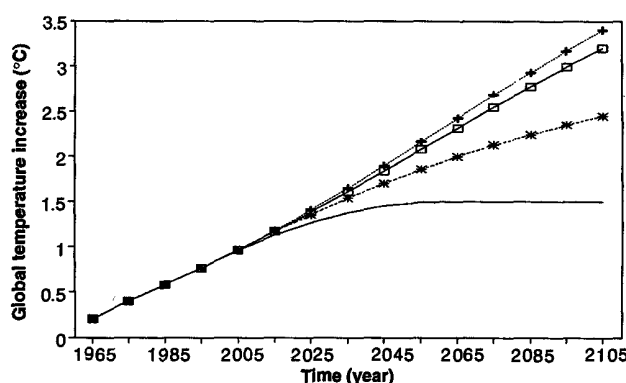
To implement policies, governments might impose carbon taxes on products producing GHG emissions. A carbon tax should be thought of as the tax (or its regulatory equivalent, say an auctionable quota) that would be necessary to raise fossil fuel and other prices sufficiently to induce economic agents to substitute away from GHG-intensive inputs and outputs. The optimal path shows a carbon tax of around \$5 per ton of carbon (or the equivalent in other GHGs) for the first control period, 1990 to 1999 (Fig. 4). The optimal carbon tax increases gradually to around \$20 per ton of carbon by the end of the next century. The carbon tax in the emissions stabilization policy rises sharply to around \$100 per ton early in the 21st century, whereas the ecological policy has extremely high carbon taxes, reaching around \$800 per ton of carbon by late in the next century. For reference, carbon taxes of \$10 and \$800 per ton of carbon represent \$7 and \$560, respectively, per ton of coal (compared to a current market price of \$30 per ton) and \$0.80 and \$64 per barrel of oil (compared to a market price of \$20 per barrel in mid-1992).

The impact of alternative policies is measured by corrected gross world output ( $Y^*$ ), which totaled about \$20 trillion in 1990. Conceptually,  $Y^*$  equals gross world product less the flow of damages from climate change less the costs of mitigation. Figure 5 shows the difference in  $Y^*$  from the no-controls path for different policies. Al-

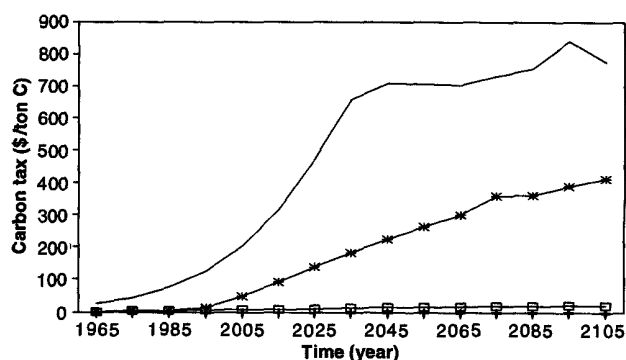
**Fig. 2.** Greenhouse-gas concentrations. Concentrations of GHGs (including  $\text{CO}_2$  and CFCs, in  $\text{CO}_2$ -equivalent basis) are estimated to double from pre-industrial levels around 2050 for the uncontrolled path (+). The optimal path ( $\square$ ) shows a small reduction in concentrations, whereas emissions stabilization (\*) leads to continued increases. Climate stabilization (—) would require declining concentrations.



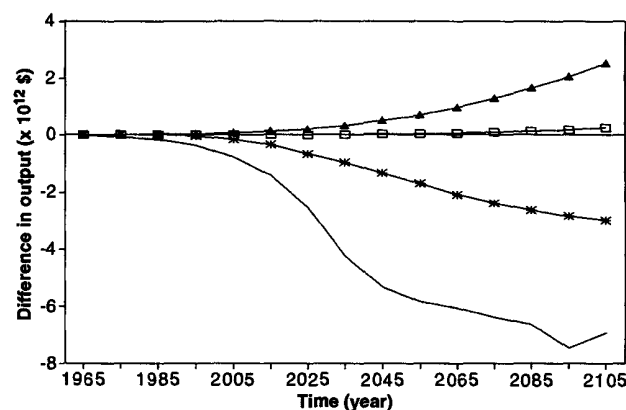
**Fig. 3.** Projected global mean temperature. According to the DICE model, global mean temperature with no controls (+) is projected to increase 3°C above 1900 levels by 2085. The optimal policy ( $\square$ ) and emissions stabilization (\*) would involve only a small reduction in global warming. The maximum feasible policy is climate stabilization (—), which shows significant warming because of the commitment in the current buildup of GHGs.



**Fig. 4.** Carbon taxes in different policies to reduce GHGs. Carbon taxes are a good index of the stringency of policies to slow global warming. A carbon tax would penalize production and consumption of fossil fuels and CFCs and encourage afforestation. Calculations indicate that the optimal carbon tax ( $\square$ ) rises from around \$5 to about \$20 per ton of carbon in 2100. Emissions stabilization (\*) and climate stabilization (—) imply sharply rising carbon taxes that would raise coal prices severalfold. Carbon tax given in 1989 U.S. dollars.



**Fig. 5.** Impact of policies on global output. The DICE model calculates the difference in economic welfare between a policy and a no-controls approach. The optimal policy ( $\square$ ) leads to slightly lower output in the first few decades, then raises output modestly after 2025. Geoengineering ( $\blacktriangle$ ) produces major benefits, whereas emissions stabilization (\*) and climate stabilization (—) are projected to be worse than inaction.



though the difference between the no-controls and the optimal policies is small, there are big stakes in the three other policies. The impact of a geoengineering solution would be quite substantial because it would reduce both mitigation costs and damages. According to these estimates, there is potential for a major waste of resources if greenhouse policies go too far. Emissions stabilization would lower world income by an amount rising to almost \$3 trillion annually by the end of the next century; climate stabilization would have an impact more than twice as large.

The present study has investigated the implications of economic growth on the environment as well as the economic impact of different environmental control strategies on the global economy. The major conclusions are the following.

First, an efficient strategy for coping with greenhouse warming must weigh the costs and benefits of different policies in an intertemporal framework. Society must balance the costs of acting prematurely against those of acting too late. The tools of optimal economic growth can be used to analyze alternative strategies.

Second, we have examined five different goals or approaches to GHG control: no control, an economic optimization, geoengineering, stabilization of emissions, and stabilization of climate. Among these five, the rank order (from a purely economic point of view) at the present time is geoengineering, economic optimum, no control, emissions stabilization, and climate stabilization. The advantage of geoengineering over other policies is enormous, although this result assumes the existence of an environmentally benign geoengineering option. The policies of no controls, the economic optimum, and emissions stabilization have impacts that are less than 1% of discounted consumption. Climate stabilization would appear enormously expensive.

Finally, it should be emphasized that this analysis has a number of important qualifications. The most important shortcoming is that the economic impact of

climate change, particularly the response of developing countries and natural ecosystems to climate change, is poorly understood at present. Furthermore, the calculations omit other potential market failures, such as ozone depletion, air pollution, and research and development, which might reinforce the logic behind GHG reduction or carbon taxes. And finally, this study abstracts from issues of uncertainty, in which risk aversion and the possibility of learning may modify the stringency and timing of control strategies. In spite of these qualifications, the optimal growth approach can clarify the scientific, economic, and policy issues that must underpin any rational decision (22).

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- 22 This research was supported by the National Science Foundation. The author is grateful for suggestions from J Ausubel, A Manne, T Schelling, M Schlesinger, S Schneider, P Waggoner, and J Weyant and for research assistance from Z Yang